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Molloy, Timothy L., Garden, Grace S., Perez, Tristan, Schiffner, Ingo, Karmaker, Debajyoti, & Srinivasan, Mandyam V. (2018)

An inverse differential game approach to modelling bird mid-air collision avoidance behaviours. In

Proceedings of the 18th IFAC Symposium on System Identification, SYSID 2018 (IFAC-PapersOnLine, Volume 51, Issue 15), IFAC (International Federation of Automatic Control), Stockholm, Sweden, pp. 754-759.

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https://doi.org/10.1016/j.ifacol.2018.09.164

An Inverse Differential Game Approach to Modelling Bird Mid-Air Collision Avoidance Behaviours*

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Abstract: In this paper, we investigate an inverse differential game approach to modelling the mid-air collision avoidance behaviours of birds. We propose a general method for estimating the cost-functional parameters of a noncooperative differential game from partial-state measurements of an open-loop Nash equilibrium. We apply the method to data of birds performing mid-air collision avoidance. Our analysis suggests that a differential game model provides a close description of the observed bird behaviours, and could provide new insights for the design of collision avoidance strategies for unmanned aircraft.

Keywords: Aerospace, Biocybernetics, Differential games, Game theory, Nash games.

1. INTRODUCTION

Game theory provides a variety of mathematical models for describing the behaviour of decision makers (or players) in conflict situations spanning fields as diverse as engineering (Basar and Olsder, 1999; Isaacs, 1965; Konstantakopoulos et al., 2016), economics (Arcidiacono et al., 2016; Bajari et al., 2007), and ecology (Hansen, 1986). Noncooperative differential games are a specialised class of game-theoretic models capable of describing the behaviour of multiple decision makers. These decision makers aim to achieve their individual objectives by interacting in realtime through a process that evolves according to a system of differential equations (Basar and Olsder, 1999; Isaacs, 1965). Noncooperative differential games have a long history as a tool for designing optimal collision avoidance manoeuvres for vehicles (Mylvaganam et al., 2017; Isaacs, 1965; Basar and Olsder, 1999). However, despite their use in the design of vehicle collision avoidance strategies, there appear to be few, if any, differential game models of collision avoidance behaviours in biological species. Inspired by recent advances in inverse techniques for estimating differential game models, in this paper we investigate an inverse differential game approach for modelling the midair collision avoidance behaviours of budgerigars (*Melopsittacus undulatus*).

Collision avoidance behaviours exhibited by biological species have the potential to inform the design of collision avoidance strategies for autonomous vehicles. Indeed, considerable effort has been directed towards studying the behaviours of animals in potential collision scenarios (Brace et al., 2016; Schiffner et al., 2016; Boardman et al., 2013; Karaman and Frazzoli, 2012). For example, Boardman et al. (2013) and Brace et al. (2016) examine the suitability of "collision cone" or effort-minimising approaches as models for the collision avoidance behaviours of bats (Myotis velifer), birds (Hirundo rustica), and fish (Danio aequipinnatus) when they are in a group. Most recently, Schiffner et al. (2016) examined the preferences of budgerigars to turn left or right, and climb or descend in order to avoid collisions in head-on encounters with other budgerigars. Nevertheless, there appear to be few, if any, examples of the use of differential games to quantitatively model the collision avoidance behaviours of biological species.

The scarcity of differential game models in the study of biological collision avoidance behaviours may in part be due to the difficulty in estimating differential games from experimental data. Indeed, the inverse differential game problem of recovering the objectives of players from Nash equilibrium solutions has only recently received attention (Molloy et al., 2017a; Rothfußet al., 2017), despite the

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^{*} This work was conducted under the Australian Research Council Linkage Project LP130100483 with the support of Boeing Research and Technology Australia.

popularity of similar inverse optimal control, inverse static game, and inverse dynamic game problems (Mombaur et al., 2010; Molloy et al., 2016; Johnson et al., 2013; Konstantakopoulos et al., 2016; Arcidiacono et al., 2016; Tsai et al., 2016; Molloy et al., 2017b). Existing inverse differential game techniques are therefore in a state of infancy and unable to explicitly handle sampled noisecorrupted partial-state measurements of the Nash equilibrium trajectories. Here, we shall extend previous treatments of inverse differential games to propose a method of inverse differential games that explicitly handles partialstate measurements of the observed Nash equilibrium. Our method is similar in spirit to the bilevel method of inverse optimal control proposed in (Mombaur et al., 2010) for studying human locomotion, and the nested inverse differential game method proposed in (Molloy et al., 2017a).

The main contributions of this paper are: (i) The proposal of a new approach to inverse differential games for estimating the cost-functional parameters of two-player differential games from partial-state measurements of an open-loop Nash equilibrium; and, (ii) The proposal of a differential game model of mid-air collision avoidance behaviour in birds. These contributions are coupled since we exploit our proposed inverse differential game approach to estimate the unknown parameters of our differential game model of bird mid-air collision avoidance behaviour. We focus on open-loop Nash equilibria since they provide a bound on the achievable value of a game and can correspond to open-loop realisations of feedback strategies (cf. (Basar and Olsder, 1999)).

The rest of this paper is structured as follows. In Section 2 we formulate the general problem of inverse differential games with an open-loop information structure and partial-state measurements. In Section 3 we propose our general method of inverse differential games and discuss its implementation. In Section 4 we propose our differential game model of bird mid-air collision avoidance and specialise our method of inverse differential games to estimate its parameters. We offer conclusions in Section 5.

2. PROBLEM FORMULATION

Let us consider a noncooperative continuous-time two player differential game with state process

$$\dot{x}(t) = f(x(t), u_1(t), u_2(t)), \quad x(0) = \bar{x} \in \mathbb{R}^n$$
 (1)

for $t \in [0, T]$ where $f(\cdot, \cdot, \cdot)$ is a possibly nonlinear function, and $u_1(\cdot) : [0, T] \mapsto \mathbb{R}^{m_1}$ and $u_2(\cdot) : [0, T] \mapsto \mathbb{R}^{m_2}$ are the controls selected by Players 1 and 2, respectively. Let us also define the cost functional for Player *i* as

$$J_{i}(x, u_{1}, u_{2}, \theta_{i}) = h_{i}(x(T), \theta_{i}) + \int_{0}^{T} g_{i}(x(t), u_{1}(t), u_{2}(t), \theta_{i}) dt$$
⁽²⁾

where $h_i(\cdot, \cdot) : \mathbb{R}^n \times \Theta_i \to \mathbb{R}$ is a terminal cost function penalising the terminal state x(T), and $g_i(\cdot, \cdot, \cdot, \cdot) : \mathbb{R}^n \times \mathbb{R}^{m_1} \times \mathbb{R}^{m_2} \times \Theta_i \to \mathbb{R}$ is a stage-cost function penalising the states and player controls. We assume that both the terminal and state cost functions for each player are parametrised by the (possibly unknown) vectors $\theta_i^* \in \Theta_i$ where $\Theta_i \subset \mathbb{R}^{M_i}$. We further assume that the players seek open-loop Nash equilibrium control trajectories u_1^{N*} and u_2^{N*} that solve the following two (coupled) optimal control problems (Basar and Olsder, 1999, p. 266):

$$\inf_{u_1} J_1\left(x^{N*}, u_1, u_2^{N*}, \theta_1^*\right)
s.t. \dot{x}^{N*}(t) = f\left(x^{N*}(t), u_1(t), u_2^{N*}(t)\right)
x^N(0) = \bar{x}$$
(3)

and

$$\inf_{u_2} \quad J_2\left(x^{N*}, u_1^{N*}, u_2, \theta_2^*\right)
s.t. \quad \dot{x}^{N*}(t) = f\left(x^{N*}(t), u_1^{N*}(t), u_2(t)\right)
\quad x^N(0) = \bar{x}.$$
(4)

Here, we let x^{N*} denote the state trajectory associated with a pair of open-loop Nash equilibrium control trajectories u_1^{N*} and u_2^{N*} .

In this paper, we consider the inverse differential game problem of estimating the parameters θ_1^* and θ_2^* of the player cost functionals from noise-corrupted partial-state measurements of the open-loop Nash equilibrium state trajectory x^{N*} given the functions $f(\cdot, \cdot, \cdot)$, $g_i(\cdot, \cdot, \cdot, \cdot)$, and $h_i(\cdot, \cdot)$. We assume that the noise-corrupted partial-state measurements are of the form

$$y(t_k) = Cx^{N*}(t_k) + w_k \tag{5}$$

for $k = 1, \ldots, K$ where $0 = t_1 < t_2 < \ldots < t_K = T$ are the sampling times, $w_k \in \mathbb{R}^{\ell}$ for $k = 1, \ldots, K$ is a zero-mean (possibly non-Gaussian) white noise process, and $C \in \mathbb{R}^{\ell \times n}$ is a matrix selecting states from $x^{N*}(t)$. We shall use our proposed method of inverse differential games to estimate the parameters of a differential game model of mid-air collision avoidance behaviour in birds.

3. INVERSE DIFFERENTIAL GAMES WITH PARTIAL STATE MEASUREMENTS

In this section, we present our proposed method of inverse differential games. We also discuss the implementation of our method for classes of differential games that can be solved using conditions for open-loop Nash equilibria.

3.1 Proposed Method of Inverse Differential Games

To propose our method of inverse differential games, let us define the functional

$$\mathcal{J}_T(x,y) \triangleq \sum_{k=1}^{K} \|Cx(t_k) - y(t_k)\|^2.$$
(6)

This functional quantifies the squared-error between a candidate state trajectory x and the sequence of partial-state noise-corrupted measurements y. Our proposed method of inverse differential games seeks to estimate the player costfunctional parameters θ_1^* and θ_2^* together with the initial state $x^*(0)$ by identifying estimates $\hat{\theta}_1$, $\hat{\theta}_2$, and \hat{x} that solve the optimisation problem

$$\inf_{\theta_1,\theta_2,\bar{x}} \mathcal{J}_T\left(x^N,y\right) \tag{7}$$

subject to

$$\begin{aligned} \dot{x}^{N}(t) &= f\left(x^{N}(t), u_{1}^{N}(t), u_{2}^{N}(t)\right) \\ x^{N}(0) &= \bar{x} \\ J_{1}\left(x^{N}, u_{1}^{N}, u_{2}^{N}, \theta_{1}\right) \leq J_{1}\left(x^{N}, u_{1}, u_{2}^{N}, \theta_{1}\right) \ \forall \, u_{1} \in \mathbb{R}^{m_{1}} \\ J_{2}\left(x^{N}, u_{1}^{N}, u_{2}^{N}, \theta_{2}\right) \leq J_{2}\left(x^{N}, u_{1}^{N}, u_{2}, \theta_{2}\right) \ \forall \, u_{2} \in \mathbb{R}^{m_{2}} \\ \theta_{i} \in \Theta_{i}, \ i = 1, 2 \end{aligned}$$

where we have used $u_i \in \mathbb{R}^{m_i}$ to denote that $u_i(t) \in \mathbb{R}^{m_i}$ for all $t \in [0, T]$.

The optimisation problem (7) involves an upper level of optimisation over the parameters and initial state, and a lower level of optimisation in the constraints to identify the open-loop Nash equilibrium corresponding to the parameters selected in the upper level optimisation. The initial state \bar{x} needs to be estimated as part of the upper level optimisation because the measurements (5) are noisy and contain only partial state information whilst the lower level of optimisation in the constraints requires completestate information. Solving the optimisation problem (7) therefore involves the nested solution of the (full-state information) noncooperative differential game (1) and (2). Since the solution of noncooperative differential games for open-loop Nash equilibria is nontrivial in general, our proposed method may not have an efficient analytic solution, and its implementation may be nontrivial.

Remark 3.1. We may also exploit measurements of the player open-loop Nash equilibrium control trajectories in our method by solving (7) with the augmented functional

$$\mathcal{J}_T(x,y) + \sum_{i=1}^2 \sum_{k=1}^K \|u_i(t_k) - z_i(t_k)\|^2$$

where

$$z_i(t_k) = u_i^{N*} + v_k$$

for $k = 1, \ldots, K$ are the control measurements corrupted by the noise process v_k . Our proposed method (7) can also be generalised to inverse differential game problems with more than two players. However, each additional player will increase the computational complexity of our proposed method (7) superlinearly by increasing the number of variables to be optimised in the upper level of optimisation whilst also increasing the complexity of identifying equilibria in the lower level of optimisation.

3.2 Implementation of Proposed Method

We propose implementing our method of inverse differential games (7) by using a gradient-based interior point optimisation routine to solve the upper level optimisation. Inverse optimal control methods (or one-player inverse differential game methods) have previously been implemented with derivative-free numerical optimisation routines to solve similar upper-level problems (cf. (Mombaur et al., 2010)). However, we have found that derivativefree routines perform poorly compared to gradient-based routines in problems with smooth parameterisations of the player cost functionals.

We also propose implementing our method (7) by replacing its lower level optimisation with the simpler problem of finding trajectories x^N , u_1^N , and u_2^N that satisfy the necessary conditions for existence of open-loop Nash equilibria given by Theorem 6.11 of Basar and Olsder (1999). In the case of linear-quadratic differential games (i.e., games with linear dynamics (1) and quadratic player cost-functionals J_i) these necessary conditions are also sufficient. However, in general the satisfaction of the necessary conditions will yield trajectories x^N , u_1^N , and u_2^N that are locally optimal solutions to the coupled optimal control problems (3) and (4). The use of necessary conditions thus allows us to find cost-functional parameters under which the measurements y may also be generated by local equilibria. Consideration of local equilibria is particularly attractive when the measurements y are from experimental demonstrations since it allows for the possibility that the players may have played sub-optimally or made mistakes in their demonstrations (see (Levine and Koltun, 2012) for a discussion of this issue in the closely related context of inverse optimal control).

The problem of finding trajectories x^N , u_1^N , and u_2^N that satisfy the necessary conditions for open-loop Nash equilibria is also simpler than solving the two coupled optimal control problems (3) and (4) since it may be formulated as a two-point boundary value problem. Here, we present this two-point boundary value problem formulation under the standard assumptions that the functions $f(\cdot, \cdot, \cdot)$ and $g_i(\cdot, \cdot, \cdot)$ are continuously differentiable in each of their arguments. Let us define the Hamiltonian functions

$$H_{i}(\lambda_{i}(t), x(t), u_{1}(t), u_{2}(t), \theta_{i}) \\ \triangleq g_{i}(x(t), u_{1}(t), u_{2}(t), \theta_{i}) + \lambda_{i}'(t)f(x(t), u_{1}(t), u_{2}(t))$$

for i = 1, 2. We recall from Theorem 6.11 of Basar and Olsder (1999) that if x^N , u_1^N , and u_2^N constitute an openloop Nash equilibrium solution to the differential game (1) and (2) for some $\theta_i^* = \theta_i \in \Theta_i$ with i = 1, 2, then there exist adjoint functions $\lambda_i(\cdot) : [0,T] \mapsto \mathbb{R}^n$ satisfying the differential equations

$$\dot{\lambda}_i(t) = -\nabla_x H_i\left(\lambda_i(t), x^N(t), u_1^N(t), u_2^N(t), \theta_i\right) \quad (8)$$

with terminal boundary condition

$$\lambda_i'(T) = -\nabla_x h_i\left(x(T), \theta_i\right)$$

for i = 1, 2. Furthermore, the player controls satisfy $\nabla = \frac{H}{2} \left(\sum_{i=1}^{N} \frac{e^{N}(t)}{2} e^{N}(t) e^{N}(t) e^{N}(t) + 0 \right) = 0$

$$\nabla_{u_i} H_i \left(\lambda_i(t), x^{**}(t), u_1^{**}(t), u_2^{**}(t), \theta_i \right) = 0$$
(9)
for $i = 1, 2$.

For many classes of differential games (involving linearquadratic games), we may rearrange (9) to write the openloop Nash equilibrium controls as

$$u_i^N(t) = r_i \left(x^N(t), \lambda_i(t), \theta_i \right)$$

where $r_i(\cdot, \cdot)$: $\mathbb{R}^n \times \mathbb{R}^n \times \Theta_i \mapsto \mathbb{R}^{m_i}$ is a function dependent on the partial derivatives of the dynamics and player cost functions. Substitution of these expressions for the control functions into the dynamics (1), and the player adjoint differential equations (8) leads to a twopoint boundary problem in $x^N(t)$, $\lambda_1(t)$, $\lambda_2(t)$ with initial conditions for the state $x^N(0)$ and terminal conditions for the player adjoint variables $\lambda_i(T)$. We may solve this two-point boundary value problem (e.g. using a shooting method or a finite difference method) for a state trajectory that satisfies the necessary conditions for an open-loop Nash equilibrium with parameters θ_1 and θ_2 , and initial state \bar{x} . These states may then be sampled to compute the objective functional in (7).

We shall now exploit our proposed method of inverse differential games to estimate a model of bird mid-air collision avoidance from experimental data.

4. MODELLING BIRD MID-AIR COLLISION AVOIDANCE BEHAVIOURS VIA INVERSE DIFFERENTIAL GAMES

In this section, we propose a differential game model of mid-air collision avoidance between two birds. We then use our method of inverse differential games (7) to estimate the parameters of the model from noise-corrupted partial-state information collected in prior experiments.

4.1 Bird Mid-Air Collision Avoidance Experiments

In previous work, experiments were conducted in which two budgerigars - Melopsittacus undulatus - were trained to fly from opposite ends of an enclosed tunnel to pressure them into a head-on encounter (Schiffner et al., 2016). The tunnel was purpose built with height 2.40m, width 1.40m, and length 21.6m. A total of 102 flights were recorded using two synchronised video cameras. The videos of a number of flights were manually processed to reconstruct the 3D positions of each bird during the encounters. This reconstruction was performed by manually labelling a point on each bird's body in each camera frame, and then using camera calibration information to project the points into 3D space. In total, this process yielded suitable position data for the 7 flights (flights discarded included those with large labelling errors due to the body points being regularly occluded).

4.2 Proposed Differential Game Model

To model the bird behaviours in these experiments, let $x_i(t) \in \mathbb{R}^6$ for $t \in [0, T]$ be Bird *i*'s position and velocity where the components $(x_i^1(t), x_i^2(t))$, $(x_i^3(t), x_i^4(t))$, and $(x_i^5(t), x_i^6(t))$ are the *x*, *y*, and *z* components of position and velocity, respectively. We consider the position and velocity of Bird *i* to evolve according to the kinematic equations of motion

$$\dot{x}_i(t) = \bar{A}x_i(t) + \bar{B}u_i(t), \quad x_i(0) = x_{i,0}$$

for $t \in [0,T]$ where $u_i(t) \in \mathbb{R}^3$ is the bird's acceleration vector, and the matrices \overline{A} and \overline{B} are given by

We write the dynamics of the game as in (1) by letting

 $f(x(t), u_1(t), u_2(t)) = Ax(t) + B_1 u_1(t) + B_2 u_2(t) \quad (10)$ where

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, A \triangleq \begin{bmatrix} \bar{A} & 0 \\ 0 & \bar{A} \end{bmatrix}, B_1 \triangleq \begin{bmatrix} \bar{B} \\ 0 \end{bmatrix}, \text{ and } B_2 \triangleq \begin{bmatrix} 0 \\ \bar{B} \end{bmatrix}.$$

We hypothesise that:

- H1) Each bird controls its acceleration $u_i(t)$ with the aim of using minimal effort to avoid collisions and reach a desired position by the terminal time T; and,
- H2) The birds perform noncooperative collision avoidance and do not coordinate their controls.

We incorporate our first hypothesis H1 into our differential game model by considering each bird to be minimising a cost functional of the form (2) with:

(i) A quadratic terminal cost given by

$$h_i(x(T), \theta_i) = (x_i(T) - q_i)' Q_i(x_i(T) - q_i),$$
 (11)

to model Bird *i*'s aim to reach a desired position $q_i \in \mathbb{R}^3$ by the terminal time T; and,

(ii) A stage-cost function given by

$$g_i(x, u_1, u_2, \theta_i) = u'_i(t)R_i u_i(t) + \delta_i(t), \qquad (12)$$

to model Bird i's aim to use minimal effort whilst avoiding collisions.

The matrix $Q_i \in \mathbb{R}^{3\times3}$ in (11) is diagonal and positive semi-definite. Similarly, the matrix $R_i \in \mathbb{R}^{3\times3}$ in the first term of (12) is diagonal and positive semi-definite to penalise large accelerations $u_i(t)$. Finally, the second term in (12) penalises close proximity to the other bird and is defined as the (weighted) inverse distance

$$\delta_i(t) \triangleq \alpha_{i,1} \left(\alpha_{i,2} \Delta_x^2(t) + \alpha_{i,3} \Delta_y^2(t) + \alpha_{i,4} \Delta_z^2(t) \right)^{-\alpha_{i,5}}$$

where $\Delta_x^2(t)$, $\Delta_y^2(t)$, and $\Delta_z^2(t)$ are the squared distances between the birds at time t in the x, y, and z directions, respectively, with the weights $\alpha_{i,j}$ for $1 \leq j \leq 5$ modelling the bird's potential preference to have different horizontal and vertical separation distances.

We incorporate our second hypothesis H2 into our model by considering each bird to be playing for a Nash equilibrium solution in which they have no incentive to unilaterally change their controls. We consider open-loop Nash equilibria since we are interested in modelling encounters between birds where mid-air collision is imminent and they must take immediate action to avoid collision (i.e., there is limited time to modify actions based on feedback). We note that open-loop Nash equilibria can also correspond to open-loop realisations of feedback strategies (cf. (Basar and Olsder, 1999)). In our model, the bird trajectories are generated with knowledge of the bird initial positions and velocities and the parameters θ_1 and θ_2 .

4.3 Parameter Estimation via Inverse Differential Games

Our proposed model of bird collision avoidance is of the form of the differential games we considered in Sections 2 and 3. The parameters of our proposed model are the diagonal elements of the matrices R_i and Q_i , the weights $\alpha_{i,1}, \ldots, \alpha_{i,5}$, and the desired final positions q_i . We collect these parameters in the vectors

$$\begin{aligned} \theta_i &= \left[R_i^{11}, R_i^{22}, R_i^{33}, Q_i^{11}, Q_i^{22}, Q_i^{33}, q_i^1, \\ & q_i^2, q_i^3, \alpha_{i,1}, \alpha_{i,2}, \alpha_{i,3}, \alpha_{i,4}, \alpha_{i,5} \right]' \in \mathbb{R}^{14} \end{aligned}$$

for i = 1, 2. We shall estimate these parameters θ_i from the position measurements collected in the experiments described in Section 4.1 using our proposed method of inverse differential games (7). Under our proposed model of mid-air collision avoidance, the position measurements correspond to noise-corrupted partial-state measurements of the bird open-loop Nash equilibrium trajectories x^{N*} . That is, the position measurements satisfy (5) with

$$C = \begin{bmatrix} \bar{C} & 0 \\ 0 & \bar{C} \end{bmatrix} \text{ where } \bar{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

and the noise w_k is due labelling errors and inaccuracies in the trajectory 3D reconstructions.

4.4 Results

We implemented our inverse differential game method (7) as described in Section 3.2. The upper level optimisation

Table 1. Errors between the experimental position measurements and trajectories predicted with our estimated differential game model.

Case No.	Root-Mean-Square Errors (mm)	
	Bird 1	Bird 2
3	25.8	41.4
5	33.7	71.9
13	8.33	10.5
15	17.1	23.3
17	8.11	15.6
27	35.9	26.8
33	26.1	16.4

in (7) was performed with the interior-point algorithm implemented in MATLAB's fmincon function, and the lower level solution of the differential game via a two-point boundary problem was achieved with the finite-difference method implemented in MATLAB's bvp4c function.

We applied our method of inverse differential games to each of the 7 experimental flights. For each flight, we estimated the parameters θ_1 and θ_2 as well as the initial states \bar{x} . We used these estimated initial states and parameters to solve our proposed differential game model to generate predicted open-loop Nash state trajectories x^N . The rootmean-square (RMS) errors between the predicted position trajectories and the position measurements for each bird and each flight are reported in Table 1.

From Table 1, we see that our differential game model with parameters estimated using our proposed method of inverse differential games (7) is able to generate predicted position trajectories with a maximum RMS error of 7.19 cm. Furthermore, only two of the predicted position trajectories (Bird 2 in Case No. 5 and Bird 2 in Case No. 3) have an RMS error greater than 3.59 cm. These two cases are plotted in Fig. 1 (Case No. 3) and Fig. 2 (Case No. 5). Visual inspection of Figs. 1 and 2 suggests that despite Case No. 3 and Case No. 5 having the highest RMS errors of our 7 cases, the predicted trajectories generated by our estimated differential game model in these cases still reasonably capture the dynamics in the data. The trajectories predicted by our estimated differential game model also provide a close visual match to the data in the 5 other experimental cases (as suggested by the lower RMS errors in the other cases).

We now examine the contribution of the estimated terminal $h_i(x(T), \theta_i)$, control $u'_i(t)R_iu_i(t)$, and separation $\delta_i(t)$ costs to the total value of the estimated cost-functionals $J_i(x^N, u_1^N, u_2^N, \theta_i)$ in Case No. 3 (we omit a full discussion due to space). In Case No. 3, the estimated control and separation costs account for 40% and 60%, respectively, of Bird 1's total cost, while the separation cost constitutes 100% of Bird 2's total cost. The large separation cost incurred by Bird 2 relative to its terminal and control costs is due to it taking no avoidance action and reaching its final position by flying in a straight line (as seen in Fig. 1). The estimated parameters θ_2 of our model capture Bird 2's preference to remain on a straight trajectory since they penalise distance from the desired terminal position more than close proximity to Bird 1 (i.e., the estimated terminal cost parameters $Q_2^{11} = 7.2, Q_2^{22} = 7.6$, and $Q_2^{33} = 21.1$ are



Fig. 1. Measured and predicted position trajectories of the two birds in Case No. 3: (a) Isometric View, and (b) Plan View. The x, y and z axes refer to positions along the width, length and height of the tunnel, respectively. Axis scales are not equal in (b).

large while the estimated separation parameter $\alpha_{2,5} = 1 \times 10^{-7}$ greatly lowers the cost of small separations).

5. CONCLUSION

We proposed a method of inverse differential games for estimating the parameters of player cost-functionals in a two-player noncooperative differential game from partialstate measurements of an open-loop Nash equilibrium. We applied our inverse differential game method to estimate the parameters of a proposed noncooperative differential game model of bird mid-air collision avoidance behaviours. Our proposed differential game model with estimated parameters provides a close representation of bird midair collision avoidance behaviour observed in previous experiments. The simplicity of our model also suggests that it is likely to be implementable on autonomous vehicles with limited sensing resources (e.g. vision sensors). Future work will extend our inverse differential game modelling approach to a larger collection of experimental data.



Fig. 2. Measured and predicted position trajectories of the two birds in Case No. 5: (a) Isometric View, and (b) Plan View. The x, y and z axes refer to positions along the width, length and height of the tunnel, respectively. Axis scales are not equal in (b).

ACKNOWLEDGEMENTS

We thank Hong Diem Vo, and Marcel Schumacher for their assistance with the experiments and generation of the bird position data. We also thank Michael Wilson for drawing our attention to the paper by Brace et al. (2016).

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